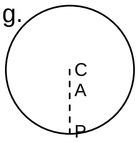
4764 MEI Mechanics 4

1(i)	$\frac{d}{dt}(mv) = mg$	B1	Seen or implied	
	$\Rightarrow \frac{dm}{dt}v + m\frac{dv}{dt} = mg$	M1	Expand	
	$\Rightarrow \frac{mg}{2(v+1)}v + m\frac{dv}{dt} = mg$	M1	Use $\frac{dm}{dt} = \frac{mg}{2(v+1)}$	
	$\Rightarrow \frac{dv}{dt} = g\left(1 - \frac{v}{2(v+1)}\right) = g\left(\frac{v+2}{2(v+1)}\right)$			
	$\Rightarrow \left(\frac{v+1}{v+2}\right)\frac{dv}{dt} = \frac{1}{2}g$	M1	Separate variables (oe)	
	$\Rightarrow \left(1 - \frac{1}{v+2}\right)\frac{dv}{dt} = \frac{1}{2}g$	E1		5
(ii)	$\int\left(1 - \frac{1}{v+2}\right)dv = \int\frac{1}{2}g\,dt$	M1	Integrate	
	$v - \ln\lvert v+2\rvert = \frac{1}{2}gt + c$	A1	LHS	
	$t = 0, v = 0 \Rightarrow -\ln 2 = c$	M1	Use condition	
	$v - \ln\lvert v+2\rvert = \frac{1}{2}gt - \ln 2$			
	$t = \frac{2}{g}(v - \ln\lvert v+2\rvert + \ln 2)$	A1		
	$v = 10 \Rightarrow t \approx 1.68$	B1		5
(iii)	As t gets large, v gets large	M1		
	So $\frac{dv}{dt} \to \frac{1}{2}g$ (i.e. constant)	A1	Complete argument	2
2(i)	$V = -mg \cdot 2a\sin\theta + \frac{\frac{1}{2}mg}{2a}(4a\sin\theta - a)^2$	B1	GPE	
		M1	Reasonable attempt at EPE	
		A1	EPE correct	
	$\frac{dV}{d\theta} = -2mga\cos\theta + \frac{mg}{2a}(4a\sin\theta - a)\cdot 4a\cos\theta$	M1	Differentiate	
	$= -2mga\cos\theta + 2mga\cos\theta(4\sin\theta - 1)$			
	$= 4mga\cos\theta(2\sin\theta - 1)$	E1	Complete argument	5
(ii)	$\frac{dV}{d\theta} = 0$	M1	Set derivative to zero	
	$\Leftrightarrow \cos\theta = 0$ or $\sin\theta = \frac{1}{2}$	M1	Solve	
	$\Leftrightarrow \theta = \frac{1}{2}\pi$ or $\frac{1}{6}\pi$	A1	Both	
	$\frac{d^2V}{d\theta^2} = 4mga\cos\theta(2\cos\theta) - 4mga\sin\theta(2\sin\theta - 1)$	M1	Second derivative (or alternative method)	
	$V''\left(\frac{1}{2}\pi\right)(= -4mga) < 0 \Rightarrow$ unstable	M1	Consider sign	
		A1	One correct conclusion validly shown	
	$V''\left(\frac{1}{6}\pi\right)\left(= 4mga\cdot\frac{\sqrt{3}}{2}(\sqrt{3})\right) > 0 \Rightarrow$ stable	A1	Complete argument	7

3(i)	Mass of 'ring' $\approx 2\pi r \delta r \rho$	B1	May be implied
	$\Rightarrow I_C = \int_0^a r^2 \cdot 2\pi\rho r\, dr$	M1	Set up integral
		A1	All correct
	$= \left[2\pi\rho \cdot \frac{1}{4}r^4\right]_0^a = \frac{1}{2}\pi a^4 \rho$	M1	Integrate
	$M = \pi a^2 \rho$	M1	Use relationship between ρ and M
	$\Rightarrow I_C = \frac{1}{2}Ma^2$	E1	Complete argument
			6
(ii)	$I_A = I_C + M\left(\frac{1}{10}a\right)^2$	M1	Use parallel axis theorem
	$= \frac{1}{2}Ma^2 + \frac{1}{100}Ma^2 = 0.51Ma^2$	E1	Convincingly shown
			2
(iii)	$I_A\ddot{\theta} = -Mg \cdot \frac{1}{10}a\sin\theta$	B1	LHS
		B1	RHS
	$\Rightarrow \ddot{\theta} = -\frac{g}{5.1a}\sin\theta$	M1	Expression for $\ddot{\theta}$
	θ small $\Rightarrow \sin\theta \approx \theta$	M1	Use small angle approximation
	$\Rightarrow \ddot{\theta} = -\frac{g}{5.1a}\theta$, i.e. SHM	E1	Complete argument and conclude SHM
	Period $2\pi\sqrt{\frac{5.1a}{g}} \approx 4.53\sqrt{a}$	F1	Follow their SHM equation
			6
(iv)	e.g. 	B1	Show PAC in straight line (in any direction)
	$mg \cdot \frac{9}{10}a = Mg \cdot \frac{1}{10}a$	M1	Moments or $(\Sigma m)\bar{x} = \Sigma mx$ (oe)
	$\Rightarrow m = \frac{1}{9}M$	A1	Method may be implied
	$I = 0.51Ma^2 + m\left(\frac{9}{10}a\right)^2$	M1	
	$= 0.6Ma^2$	E1	Convincingly shown
			5
(v)	KE $= \frac{1}{2}I\omega^2 = \frac{1}{2}(0.6Ma^2)\omega^2$	M1	Attempt to find KE
	$= 0.3Ma^2\omega^2$	A1	
	$C \cdot n \cdot 2\pi = 0.3Ma^2\omega^2$	M1	Work-energy equation
		A1	Correct equation
	$\Rightarrow C = \frac{0.3Ma^2\omega^2}{2n\pi}$	A1	
			5

4(i)	At terminal velocity, $\Sigma F = 0 \Rightarrow k \cdot 60^2 = 90g$	M1	Equilibrium of forces	
	$\Rightarrow k = \frac{1}{40}g$	E1	Convincingly shown	
				2
(ii)	$90v\frac{\mathrm{d}v}{\mathrm{d}x} = 90g - \frac{1}{40}gv^2$	M1	N2L	
		A1		
	$\int \frac{90v}{90g - \frac{1}{40}gv^2}\,\mathrm{d}v = \int \mathrm{d}x$	M1	Separate and integrate	
	$-\frac{1800}{g}\ln\left\|90g - \frac{1}{40}gv^2\right\| = x + c_1$	A1	LHS	
	$90g - \frac{1}{40}gv^2 = Ae^{-\frac{gx}{1800}}$			
	$v^2 = \frac{40}{g}\left(90g - Ae^{-\frac{gx}{1800}}\right)$	M1	Rearrange, dealing properly with constant	
	$x = 0, v = 0 \Rightarrow A = 90g$	M1	Use condition	
	$v^2 = 3600\left(1 - e^{-\frac{gx}{1800}}\right)$	E1	Complete argument	
				7
(iii)	WD against $R = \int_0^{1800} kv^2\,\mathrm{d}x$	B1		
	$= \int_0^{1800} 90g\left(1 - e^{-\frac{gx}{1800}}\right)\mathrm{d}x$			
	$= \left[90g\left(x + \frac{1800}{g}e^{-\frac{gx}{1800}}\right)\right]_0^{1800}$	M1	Integrate	
	$= 162000(g + e^{-g} - 1)$	A1		
	$x = 1800 \Rightarrow v^2 = 3600(1 - e^{-g})$	B1		
	Loss in energy			
	$= 90g \cdot 1800 - \frac{1}{2} \cdot 90 \cdot 3600(1 - e^{-g})$	M1	GPE	
		M1	KE	
	$= 162000(g + e^{-g} - 1) =$ WD against R	E1	Convincingly shown (including signs)	
				7
(iv)	$v = 60\sqrt{1 - e^{-g}} \approx 59.9983$	B1		
				1
(v)	$90\frac{\mathrm{d}v}{\mathrm{d}t} = 90g - 90v$	M1	N2L	
		A1		
	$\int \frac{\mathrm{d}v}{g - v} = \int \mathrm{d}t$ $\left[\text{or } \int_{59.9983}^{10} \frac{\mathrm{d}v}{g - v} = \int_0^t \mathrm{d}t\right]$	M1	Separate and integrate	
	$-\ln\|g - v\| = t + c_2$	A1		
	$t = 0, v = 59.9983 \Rightarrow c_2 = -3.91598$	M1	Use condition (or limits)	
	$v = 10 \Rightarrow t = -\ln 0.2 + 3.91598$	M1	Calculate t	
	≈ 5.53 s	A1		
				7